

RiskRank: Measuring interconnected risk[☆]

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Abstract

This paper proposes RiskRank as a joint measure of cyclical and cross-sectional systemic risk. RiskRank is a general-purpose aggregation operator that concurrently accounts for risk levels for individual entities and their interconnectedness. The measure relies on the decomposition of systemic risk into sub-components that are in turn assessed using a set of risk measures and their relationships. For this purpose, motivated by the development of the Choquet integral, we employ the RiskRank function to aggregate risk measures, allowing for the integration of the interrelation of different factors in the aggregation process. The use of RiskRank is illustrated through a real-world case in a European setting, in which we show that it performs well in out-of-sample analysis. In the example, we provide an estimation of systemic risk from country-level risk and cross-border linkages.

Keywords: systemic risk, aggregation operators, network analysis, Choquet integral

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1. Introduction

The current financial crisis has stimulated research on systemic financial risks. This has led to several contributions for measuring interconnectedness and contagion risk, as well as for estimating probabilities of systemic distress events. Yet, these two types of models have so far been built in isolation. This paper proposes RiskRank as a measure of connected risk by joining risk likelihoods and impacts.

The literature on systemic risk measurement has evolved along two dimensions [7]: cyclical and cross-sectional systemic risk. These two dimensions accentuate the need for modeling not only individual financial components, be they economies, markets or institutions, but also interconnectedness among them and their system-wide risk contributions. To this end, analytical tools and models provide ample means for two types of tasks: (i) early identification of vulnerabilities and risks, and (ii) early assessment of transmission channels of and a system's resilience to shocks. While the first task is usually tackled with early-warning indicators and models to derive a probability of a systemic crisis (e.g., Alessi and Detken [1]), macro stress-testing models and contagion and spillover models provide means to assess the resilience of the financial system to a wide variety of aggregate shocks (e.g., Castrén et al. [8]) and cross-sectional transmission of financial instability (e.g., IMF [19]), respectively. RiskRank aims at measuring both of these two dimensions concurrently.

In the vein of two strands of systemic risk measures, these can also be viewed from the perspective of various approaches to aggregating information. Early-warning models tend to focus on the aggregation of multiple indicators into a meaningful measure of cyclical systemic risk, which oftentimes takes the form of distress probabilities (e.g., Lo Duca and Peltonen [26]). Further, the literature has also provided various approaches for aggregating multiple models in order to assure more robust model output (e.g., Holopainen and Sarlin [18]). Likewise, a large share of the literature on cross-sectional systemic risk has focused on network-based measures of interconnectedness and connectivity (e.g., Billio et al. [6], Peltonen et al. [34]). RiskRank provides a centrality measure for networks, but goes beyond link-based centrality by also accounting for materialization probabilities (or node importance).

This paper puts forward RiskRank as a measure of interconnected risk. While focusing on systemic risk, the approach is general-purpose in nature by applying to any type of risk that exhibits individual materialization probabilities (i.e., risk levels of components) and impact measures (i.e., interlinkages among components). In line with the literature on aggregation operators, we put forward a framework motivated by the Choquet integral as a means to aggregate risk levels to system-wide vulnerability by also accounting for the size of interlinkages across the components of the system (be they economies, markets or institutions). Hence, this can also be seen as a network-based centrality measure that also accounts for node importance (i.e., risk levels). This provides nothing else than a likelihood of a systemic event at all levels of the system, ranging from re-calculated risk at the lowest levels to aggregated risk at the highest level. In this paper, we illustrate the use of RiskRank from country-level early-warning models to connected individual and system-wide risk. While being targeted at systemic financial risk, this flexible tool is easily adaptable to measuring any connected risk.

The rest of the paper is structured as follows. Section 2 discusses systemic risk measurement and introduces aggregation operators and centrality measures, particularly with a view to systemic risk. In Section 3, we motivate and describe the modification of the general form of Choquet integral into the Risk Rank measure and discuss its most important features and use scenarios. Section 4 presents the application of the RiskRank to the case of European systemic risk. Finally, we conclude in Section 5.

2. Measuring Systemic risk: A synthesis

To quantify systemic risk, we need a broad toolbox of models to measure and analyze system-wide threats to financial stability. In the vein of standard risk analysis, we disentangle the topic of systemic risk into two tasks: probability and impact. While assigning probabilities to events aims at ranking individual risks and vulnerabilities as per intensity (i.e., tasks of early-warning models), assessing the severity or impact of an event complements by modeling transmission channels and quantifying

losses given their materialization. This accentuates the need for modeling not only the likelihood of a distress event p_i^t in time t for entity i , be they economies, markets or institutions, but also system-wide importance by accounting for interconnectedness and other types of transmission channels m_{ij}^t between each entity i and all other entities j at time t .

This section discusses the role of systemic risk analysis from the viewpoint of the cyclical and cross-sectional dimensions. We discuss the two strands of literature for systemic risk analysis, as well as motivate the need for a general-purpose approach for joining the two strands.

2.1. Systemic risk models

Broadly speaking, tools and models can be divided into those for early identification and assessment of systemic risks. ECB [12] provides a mapping of tools to the following three forms of systemic risk: (i) early-warning models, (ii) contagion and spillover models, and (iii) macro stress-testing models.

Cyclical systemic risk. The first form of systemic risk focuses on the unraveling of widespread imbalances and is illustrated by a thorough literature on the presence of risks, vulnerabilities and imbalances in banking systems and the overall macro-financial environment prior to historical financial crises. This resembles Kindleberger’s [22] and Minsky’s [30] financial fragility view of a boom-bust credit or asset cycle. Hence, the subsequent abrupt unraveling of the imbalances may be endogenously or exogenously caused by idiosyncratic or systematic shocks, and may have adverse effects on a wide range of financial intermediaries and markets in a simultaneous fashion. Early and later empirical literature alike have identified common patterns in underlying vulnerabilities preceding financial crises (see, e.g., Kaminsky et al. [20] and Reinhart and Rogoff [38]).

First, by focusing on the presence of vulnerabilities and imbalances in an economy, early-warning models can be used to derive probabilities of the occurrence of systemic financial crises in the future (e.g., Alessi and Detken [1] and Lo Duca and Peltonen [26]). These models use a set of vulnerability and risk indicators to identify whether or not an economy is in a vulnerable state. The outputs of such models mostly take the form of a probability of a crisis within a specific time horizon and are monitored with respect to threshold values. Hence, this provides us a probabilities of crisis p_i^t in time t for entity i , where entities may be economies, markets or institutions, but does not provide information about the potential impact of the individual entities on others. Typical methods used in early-warning models include logistic models Lo Duca and Peltonen [26] and machine learning Holopainen and Sarlin [18].

Cross-sectional systemic risk. The second type of systemic risk refers to two types of models for measuring the cross-sectional dimension. Macro stress-testing models provide means to assess the resilience of the financial system to a wide variety of aggregate shocks, such as economic downturns (e.g., Castrén et al. [8] and Hirtle et al. [17]). These models allow policymakers to assess the consequences of assumed extreme, but plausible, shocks for different entities. The key question of macro stress-testing is to find the balance between plausibility and severity of the stress scenarios such that they are plausible enough to be taken seriously and severe enough to be meaningful (e.g., Alfaro and Drehmann [2] and Quagliariello [36]). Third, contagion and spillover models can be employed to assess how resilient the financial system is to cross-sectional transmission of financial instability (e.g., IMF [19]). Hence, they attempt to answer the question: With what likelihood, and to what extent, could the failure of one or multiple financial intermediaries cause the failure of other intermediaries? Accordingly, this line of work provides information on system-wide importance by accounting for interconnectedness and other types of transmission channels m_{ij} between each entity i and all other entities j . Yet, this provides little information on the likelihood of individual entities being distressed.

Another type of cross-sectional systemic risk refers to a widespread exogenous aggregate shock that has negative systematic effects on one or many financial intermediaries and markets at the same time. These types of aggregate shocks have empirically been shown to co-occur with financial instabilities (see, e.g., Gorton [13] and Demirgüç-Kunt and Detragiache [10]). An example of such an event is the collapse of banks during recessions due to the vulnerability to economic downturns. The third

form of systemic risk is contagion and spillover, which usually refers to an idiosyncratic problem, be it endogenous or exogenous, that spreads in a sequential fashion in the cross section. The cross-sectional transmission of financial instability has been empirically shown by a large number of studies (e.g., Upper and Worms [42] and van Lelyveld and Liedorp [43]). For instance, episodes of financial instabilities have been shown to relate to the failure of one financial intermediary causing the failure of another, which initially seemed solvent, was not vulnerable to the same risks and was not subject to the same original shock as the former. It is worth noting that contagion refers to a situation when the initial failure is entirely responsible for subsequent ones, whereas the term spillover is commonly used when the causal relationship is not found or cannot be tested (see, e.g., ECB [12]). A recent approach presented in Tarashev et al. [41] makes use of the Shapley index developed originally for problems of game theory. In the context of games, the Shapley index measures the average contribution of a player that he/she individually generates to the group of players as a whole. As it is described by Tarashev et al. [41], this can be naturally translated in the context of systemic risk analysis to the decomposition of various measures of system-wide risks into the systemic importance of individual entities. Lee et al. [25]

Joining the two dimensions of systemic risk. For the analysis of systemic risk, this provides a standard set-up as in any type of risk analysis: The level of risk can be calculated as the product of the probability that individual distress occurs p_i^t (e.g., p_i^t that bank i fails in quarter t) multiplied with the severity of that event for other entities through their interconnectedness m_{ij}^t (e.g., the impact of bank i on other banks j in quarter t). The literature combining these issues is scarce. A starting point has been provided by Minoiu et al. [29] and Rancan et al. [37] by using interconnectedness measures as predictors of crises. Puliga et al. [35] finds, in the case of building the network based on Credit Default Swaps contracts, that systemic risk level estimation, focusing on the time period before and after 2008, only increases if macroeconomic indicators are incorporated in the construction of the network. Yet, this provides little information on the vulnerability of one entity and its impact on others. In this vein, Peltonen et al. [33] have explicitly modeled the vulnerability of one bank as a function of the vulnerability of its neighbors through tail-dependence networks. While being a starting point, this provides no structured approach to accounting for both dimensions simultaneously.

2.2. Systemic risk aggregation

In order to estimate systemic risk, we most often rely on various approaches for aggregating information, not the least in the case of the above mentioned two dimensions of risk: multiple indicators into risk levels and measures of interconnectedness into network centrality. For a more formal view to aggregation, the value is usually obtained so that it provides a sufficient representation of the original set and satisfies a number of predefined requirements related to the underlying problem. For this purpose, different functions, termed as aggregation operators or functions, can be defined that perform the task of producing this representative value. In the following, the definitions will be formulated for the case of aggregating values from the most common $[0, 1]$ interval (which will also later on be the used range). An aggregation operator on n arguments is a function $f : [0, 1]^n \rightarrow [0, 1]$, which satisfies the following properties Beliaikov et al. [4]:

- boundary condition: if all the aggregated values are 1's (0's), then the value of f is 1 (0);
- monotonicity: if $(x_1, \dots, x_n) \leq (y_1, \dots, y_n)$, then $f(x_1, \dots, x_n) \leq f(y_1, \dots, y_n)$

Although these two basic properties are satisfied by a wide class of functions, in most of the use cases additional properties are required. For instance, while the product of numbers on the $[0, 1]$ interval is an aggregation function, it suffers from the fact that the aggregated value is smaller than the minimum of the original values. To overcome this issue, a subclass of aggregation functions can be used Grabisch et al. [16]: f is an averaging function if $\min(x_1, \dots, x_n) \leq f(x_1, \dots, x_n) \leq \max(x_1, \dots, x_n)$. The application of averaging functions in economics, specifically in systemic risk and network analysis,

is mainly restricted to the use of a handful of specifications, such as the minimum, maximum, weighted mean, and the most commonly used arithmetic mean. The importance of the arithmetic mean stems from the fact that it is the value with minimum sum of squared deviations from the original aggregated numbers, a crucial property used in many statistical methods. In different contexts and applications, one may require the aggregated value to satisfy a criterion different from minimizing the sum of squared deviations. For instance, we can look at the overall financial state of a population by employing different aggregation techniques, such as minimum, maximum, median and average income. This section looks at the two dimensions of systemic risk from the view of the aggregation procedures involved, as both components of systemic risk are most often estimated using different aggregation procedures related to indicators and network structures.

Risk indicators into probability. In the vein of the previous section on cyclical systemic risk, early-warning indicators are utilized in various ways to obtain an estimate of risk. This is essentially nothing else than an aggregation procedure. Herein, we categorize them into three classes and view them from the perspective of aggregation operators. The first class of models, denoted the signaling approach, monitors individual indicators and identifies risk when an indicator value exceeds a predefined threshold. Likewise, the multivariate signaling approach monitors concurrently a set of indicators via a performance-weighted average of several indicators, and monitors threshold exceedances. This involves either no aggregation at all or a simple weighted average. The second class of models makes use of different statistical and machine learning approaches to estimate optimal weights for combining indicators into a final probability. Some of these approaches rely on and result in a linear aggregation (i.e., weighted average) of the indicator values in the modeling process, with linear discriminant analysis as one example. However, in most cases we obtain some non-linearity in the aggregations, such as even in the simple logistic regression. The number of different approaches employed for aggregating indicators into a probability is large, such as classification trees Duttagupta and Cashin [11], logistic regression Lo Duca and Peltonen [26], artificial neural networks Sarlin [40] and k -nearest neighbors Holopainen and Sarlin [18]. The third class of estimation models relies on the combination of methods from the second class through ensemble learning. Typical procedures to generate ensemble models rely on estimating a large number of individual models that are to be aggregated into a final one (for further details on ensembles see Holopainen and Sarlin [18]). Model aggregation could happen at two different levels: aggregating binary model output via a majority vote (i.e., median) or aggregating probabilistic model output through arithmetic or weighted means. To this end, one can conclude that each and every multivariate approach for deriving early-warning models relies on an aggregation of indicators.

Interlinkages into centrality. Drawing upon the literature on interconnectedness and networks, the literature has obviously proposed a large number of measures that aim at revealing network properties. Considering a single node in a network, a traditional way is to look at different centrality measures that can help to understand the role of a specific node in the network. Centrality measures play a key role in systemic risk analysis, as they provide means to indicate interconnectedness or overall importance of a node. Beyond standard measures from graph theory, the literature also consists of more customized measures, such as DebtRank Battiston et al. [3] as way of identifying systematically important nodes in a financial system using a feedback centrality measure. Instead of aggregating interconnections among entities, one may also approach the problem from the perspective of decomposing risk contributions, such as the Shapley index approach of Tarashev et al. Tarashev et al. [41]. Their proposal uses the Shapley index to estimate the systemic risk contribution of entities utilizing a characteristic function defined on all the subsets of the system of entities. They propose to use various characteristic measures, mainly focusing on Value-at-Risk (VaR) and Expected Shortfall (ES), but also note that any risk measure can be used as the basis of calculating the Shapley index.

The purpose of various centrality measures is most often to describe the nodes of a network from two perspectives: (i) how they affect other (e.g., neighboring) nodes directly or indirectly, and (ii) how they are affected by other nodes. These two different perspectives can be measured for example

with the in and out-degree (strength) of a node. More complex measures focus on the centrality of the nodes from different perspectives: importance of a node for connecting others (betweenness centrality) and how distant it is on average from all the other nodes (closeness centrality). From a general point of view, most centrality measures essentially serve as an aggregation measure: for every node, we collect specific information (e.g., in-degree or shortest path) with respect to a subset of the other nodes (e.g. neighboring nodes or the set of all nodes), and combine the information into a single “average value”. This corresponds to the general notion of aggregation of summarizing a set of numerical values into a single number that conveys some predefined characteristics of the original set of values. According to this, a centrality measure is a function, that assigns a real value for every node of a network given the adjacency matrix, A . An important question is what information (which sub-matrix of the adjacency matrix) should be considered in obtaining a required characteristic of a node. For example, in-degree of a node utilizes only the column corresponding to the node, degree centrality makes use of one row and one column (in and out-degrees of the node), while we potentially need the whole matrix to calculate the shortest path or closeness centrality. Additionally, different measures use different functions to aggregate the individual values into a final (centrality) measure, such as the sum of the values, the minimum operator or a combination of these two. Finally, we can notice that formally almost all the centrality measures used in practice satisfy the property required from a well-defined aggregation procedure: monotonicity. For instance, if we increase the weight of a link in the network, the degree centrality for a node or the shortest possible distance between two nodes can never decrease.

Most of the measures in the literature rely only on link values when determining centrality, without considering values associated to the nodes themselves. The purpose in the following section is to focus on the aggregation procedure in the context of network centrality measures and specify an operator that can incorporate both node and link values in calculating different node and network characteristics.

3. RiskRank as an aggregation operator

This section describes our approach for concurrent measurement of interconnected risk, particularly cyclical and cross-sectional systemic risk. The considered system is represented as a directed graph, which is based on a hierarchical decomposition of the system into an interconnected network of the involved actors. In our model, actors can refer to financial institutions, financial systems, countries, individual banks, etc. As discussed in the previous section, there exists numerous approaches to analyze networks focusing on the importance or centrality of individual nodes and the level of interconnectedness to measure some generic attributes of the network. To represent and assess the two discussed dimensions of systemic risk, in the following we describe how to utilize aggregation functions to combine information regarding both the likelihood of individual risk and the interconnectedness within the network to assess systemic risk.

To present our approach for measuring systemic risk, we use the following notations throughout this section. The system is represented as a network with a hierarchical structure. The highest level (level 0) of the hierarchy consists of a single node, S , representing the systemic risk. The first level consists of the main components of the system, $S_{1,1}, S_{1,2}, \dots, S_{1,n}$ (for example countries or different financial sectors in a country). The n nodes form a complete sub-network (there is a directed link from every node to all the other nodes), and additionally all of them are connected to node S . The second level consists of n complete and pair-wisely disjoint sub-networks, each connected only to a single node from the first level in the same manner as the first level nodes are connected to S . According to this scheme, when creating a new level of the hierarchy, a complete sub-network is created for each node in the previous level. Additionally, there is a numeric value associated to every node and link in the network: the likelihood of risk in the corresponding component of the system as the node values and a measure of impact of the starting node on the end node as the link weight. In the following, S_i will denote the number of nodes on level i , S_i^j denotes the number of nodes in the $i + 1$ th level complete sub-network corresponding to node j from level i , i.e. $S_{i+1} = \sum_{j=1}^{S_i} S_i^j$, c_k and $l_{k,j}$ will denote the value associated to node k and the weight associated to the link between nodes k and j , respectively.

Finally, as we are interested in the changes taking place in the system, the network is considered at different time-points with the same structure but different node and link values.

3.1. From aggregation operators to RiskRank

As we discussed above, the main goal of our model is to provide a measure of systemic risk, and at the same time estimate the level of vulnerability in different components of the system; as the system as a whole itself is represented as a node, the task is to estimate the value of a node in a future time-point given the previous values of the nodes and links. The basic idea is that the state of a component of a system (a node in the network), at least in a short time period, is determined by the previous state of the given component and the ones which directly impact it. This problem can be reformulated as applying some kind of aggregation procedure to summarize the values in a node and its neighbors to predict a future value of a node with the natural choice being an averaging function. The most straightforward solution, which can be seen as a type of degree centrality, would be obtained as the sum of the incoming links in the network. Although this measure encompasses an important piece of information (the more a component of a system is impacted by others, the more likely it is that problems can spread through its connections), it does not make use of the additional information on the values associated to the nodes. Using the weighted average for example would calculate the impact-weighted risk measure value as a generalization of the degree centrality; the original definition is obtained in case of all the node values equal 1. To improve this measure, one could potentially consider the Ordered Weighted Average (OWA) operator introduced by Yager [44]. This averaging function reorders the values to be aggregated into decreasing order and the associated weights are assigned to the values based on this order. OWA includes, as a special case, the min, max, weighted average and many of the most used averaging functions offering more freedom in determining the structure of the aggregation process.

Still, there can be many applications where the features of OWA are not sufficient to appropriately model the underlying phenomena. To explain the most important information overlooked with the traditional approaches, one can consider aggregation as a decision making tool: evaluating a decision alternative by considering several criteria with different importance. In many problems, the considered criteria are not independent from each other, but rather a positive/negative performance on a criterion increases/decreases the importance of other criteria. A typical example is the case of medical diagnosis: in order to assess a patient's status, the presence of different symptoms is checked. While the presence of different symptoms individually does not necessarily imply the high risk of a serious disease, the simultaneous appearance of the symptoms indicates high risk. This interrelated nature of decision criteria in various applications led to the utilization of Choquet-integral based aggregation in decision making literature Labreuche and Grabisch [23]. In the problem of estimating systemic risk using a network approach, this issue can be relevant as a component of the system (the decision alternative) is not only affected by a related component (criterion) directly, but also indirectly through the joint effect of two or several connected components (interrelated criteria).

Before discussing the specific use of aggregation to obtain network centrality measures to estimate systemic risk, we introduce the notion of discrete Choquet integral as a sufficiently general averaging operator to represent interdependencies and non-linear behavior. As we attempt to model the joint effect of different components beyond considering individual ones (which is a special case of a joint effect of one component), as the basis of the aggregation, an initial function is needed to represent these joint effects. In the most complex case, every possible subset of the considered components (the power set) need to be accounted for. This representation is based on the construct of a monotone or fuzzy measure [31]:

Definition 1. A monotone measure μ on the finite set $N = \{1, 2, \dots, n\}$ is a set function $\mu : P(N) \rightarrow [0, 1]$ (where $P(N)$ is the power set of N) satisfying the following two conditions:

- $\mu(\emptyset) = 0, \mu(N) = 1$;
- $A \subseteq B$ implies that $\mu(A) \leq \mu(B)$.

One of the most general formulations utilizing a fuzzy measure representation in the aggregation process is modeled by the discrete Choquet integral [9, 28]. To formulate the definition, we suppose that there are n number of criteria to be considered (connected system components in our application), c_1, \dots, c_n , based on which an evaluation is performed, which results in the corresponding x_1, \dots, x_n importance values (the impact of the components on the considered central component).

Definition 2. A discrete Choquet integral with respect to a monotone measure μ is defined as

$$C_\mu(x_1, \dots, x_n) = \sum_{i=1}^n (x_{(i)} - x_{(i-1)}) \mu(C_{(i)})$$

where $x_{(i)}$ denotes a permutation of the x_i values such that $x_{(1)} \leq x_{(2)} \leq \dots \leq x_{(n)}$ and $C_{(i)} = \{c_{(i)}, c_{(i+1)}, \dots, c_{(n)}\}$.

Although it is not straightforward to see for all the cases, but all the previously mentioned averaging functions, specifically the OWA and consequently the weighted average, the minimum and maximum, are all special cases of this general definition with an appropriate choice of fuzzy measure. As a consequence of the complexity with regard to the need of estimating potentially 2^n coefficients in the fuzzy measure, the general Choquet integral has started to gain popularity in different applications only in recent years [32]. As it was pointed out, the crucial aspect of the Choquet-integral that makes it attractive in many applications compared to the traditional averaging functions is the capability of modeling interaction in the aggregation process. In the general multi-criteria formulation of the application of Choquet-integral, this can be translated to the interrelation of criteria: an alternative is evaluated differently based on criterion A in the simultaneous presence of criterion B than considering only A without B . In the proposed application, the risk level of an entity (country, bank) will be estimated based on its present risk level and its connection to other entities.

To ease on the complexity of estimating the joint effect of all the considered criteria (connected components in the system) but still moving beyond the simple representation of assuming independence, one can consider only the joint effect of specific subsets and assume that the others are negligible. A straightforward approach widely discussed in the literature is the case of 2-additive Choquet integral. In this formulation, only the pairwise interaction between criteria (the joint effect of two connected components) is considered additionally to the individual effects. The Choquet integral in this case can be formulated as (see Grabisch [14]):

$$C_\mu(x_1, \dots, x_n) = \sum_{i=1}^n (v(c_i) - \frac{1}{2} \sum_{I(c_i, c_j) > 0} I(c_i, c_j) \min(x_i, x_j) + \sum_{I(c_i, c_j) < 0} |I(c_i, c_j)| \max(x_i, x_j)).$$

The interaction measure $I(c_i, c_j)$ can be defined by transforming the measure for pairs into the $[-1, 1]$, a detailed discussion can be found for example in Marichal [27]. As it was pointed out for example in Keeney and Raiffa [21], the interaction index and the importance of a criteria can be interpreted in the context of game theory using traditional utility theory, and the formulas can be derived on that foundation as a form of the Shapley index. The formula can be derived by using the concept of the Möbius transformation of a fuzzy measure Grabisch and Roubens [15]. The Shapley-index is defined as

$$v(x_i) = \sum_{K \subset X \setminus i} \frac{(n - |K| - 1)! |K|!}{n!} (\mu(K \cup x_i) - \mu(K))$$

The Shapley index can be interpreted as the average contribution of the node i for all the possible subsets of the points including i , moreover it takes its value from the $[0, 1]$ interval and $\sum_{i=1}^n v_i = 1$. In our analyzed problem this translates to the overall impact a node has on another node, directly (through a link) or indirectly (as a consequence of interaction with another node). For example, in the simplest case, a node is only connected to the central/analyzed node through a direct link, with not other path between the two nodes. In this case, the only non-zero term in calculating the Shapley

index will be when $K = \emptyset$, and the formula is simplified to $\mu(x_i)/n$. For our purpose, the important issue is to understand the differences between the types of interaction represented by the terms in the formula above, and how interaction can be translated to the case of our network representation and systemic risk analysis. In our 2-additive model, additionally to the individual importance of a node connected to a central node to be assessed, we want to account for the joint effect of two nodes on the central node. In terms of the network, it means that we consider paths with length 2 that end in the central node with positive joint effect, and every other path or other subset of edges is considered with 0 interaction value. This corresponds to the case of no interaction or $I(c_i, c_j) = 0$. Negative interaction in the sense of the Choquet integral represents the cases of disjunctive effects, which are not present in our model: an increase in one node value will affect the central node also indirectly through the path including the other node. This implies that in our aggregation process, we consider only positive interaction values in the form of the product of the . Additionally, in our application the minimum operator contradicts the intuitive idea of risk spreading throughout the network along the paths; while a positive interaction indicates conjunctive behavior, using the minimum operator, an increase in the higher node value will not affect the joint effect of the two nodes. For this reason, we further modify the above formula by replacing the minimum operator by the multiplication operator.

3.2. RiskRank

To see the correspondence between the proposed Choquet integral and the Shapley index approach of Tarashev et al. [41], we first note that the most general form of the Choquet integral requires the fuzzy measure to be specified on all the subsets of the set of considered entities. In this sense, the fuzzy measure can be seen as an example of the characteristic measure referred to by Tarashev et al.. However, in the general case the calculation of the Choquet-based aggregation cannot be simplified into a function of the Shapley index of the individual entities. To be able to utilize the Shapley index, we can restrict the measurable interlinkages to pairs of individual entities, meaning that we would not define the risk contribution of subsets with cardinality higher than 2. While this is a simplification compared to a full utilization of the Choquet integral and the Shapley index approach, it allows for a natural representation in the form of a network and ensures the (2-)additivity of the proposed measure.¹ Additionally, our approach allows for the decomposition of the individual contribution of an entity further into direct and indirect effects. This offers a deeper understanding of the risk structure within the system. At the same time, additionally to an interconnectedness measure, we are also concerned with incorporating an individual risk level of the components under the analysis.

Based on the above discussion, a function resembling the structure of the discrete 2-additive Choquet integral will be used to aggregate values in a network to estimate systemic risk. For risk level (node value) x_i and interlinkage $I(c_i, c_j)$ between nodes i and j combining the interlinkage values between node i and the target node and between node i and j , RiskRank is defined as

$$RR(x_1, \dots, x_n) = \sum_{i=1}^n \underbrace{\left(v(c_i) - \frac{1}{2} \sum_j I(c_i, c_j) \right) x_i}_{\text{Direct effect of component } i} + \sum \underbrace{I(c_i, c_j) \prod (x_i, x_j)}_{\text{Indirect effect of component } i} .$$

The components of the formula express the two ways a node affects another one. The RiskRank function estimates risk level for a specific node. Accordingly, the notation $C_\mu^{S_t}$ should be used in general as the RiskRank of node S_t , but in the following, we will not use the indexing unless it is of importance for which node the RiskRank is calculated. RiskRank is calculated and interpreted slightly differently for the central node S , and any other nodes in the network; in the following we discuss the differences between the two cases.

¹As we will see in Section 4, the indirect effects (i.e. the effect resulting from the interconnectedness of two entities) are in general negligible compared to direct effects.

One important use of the RiskRank measure in analyzing the described hierarchical network is to assign a value to the node on the top level, S , representing the level of systemic risk. In this case, as this specific node does not have an initial value, the RiskRank formula can be straightforwardly applied: the nodes that are either connected to S or there is a path of length 2 from the node to S are considered in the calculations. In case of nodes from the second level of the hierarchy, as they are only connected to a single node on the first level, there is only one interaction term, while for the nodes in the first level, as they form a complete sub-network, there are $t - 1$ interaction terms for every node where t is the number of nodes on the first level. The final value for the node S provides an estimation of the likelihood that a system-wide risk is present in the network.

As for the other nodes in the network, usually we have a node value (risk level) assigned before performing the aggregation process, for example based on market estimations. According to this, additionally to their incoming links, we need to account for this observed node value in the aggregation process. As a straightforward solution, this can be done by calculating the weighted average of this node value and the value obtained from the RiskRank function based on the connections of the node in the network. Alternatively, to keep a uniform formalism, we can use the RiskRank equation without any additional aggregation by introducing a new link in the network: a self-loop for the evaluated node. The link weight is determined based on the overall exposure of the corresponding system component to the elements of the system; the measure of exposure can be different in different applications. The interaction of the evaluated node and other nodes is set as 0, in order to prevent the additional link to contribute to indirect effect as part of paths of length 2 starting from a neighboring node. We can write the formula as

$$\begin{aligned}
 RR_c(x_1, \dots, x_n, x_c) &= \sum_{i=1}^{n+1} \underbrace{\left(v(c_i) - \frac{1}{2} \sum_j I(c_i, c_j) \right) x_i}_{\text{Direct effect of component } i} + \sum_{i,j} \underbrace{I(c_i, c_j) \prod(x_i, x_j)}_{\text{Indirect effect of component } i} \\
 &= \underbrace{v(c)x_c}_{\text{Individual effect of component } c} + \sum_{i=1}^n \underbrace{\left(v(c_i) - \frac{1}{2} \sum_{j \neq i} I(c_i, c_j) \right) x_i}_{\text{Direct effect of component } i \text{ on } c} \\
 &\quad + \underbrace{\sum_i \sum_{j \neq i} I(c_i, c_j) \prod(x_i, x_j)}_{\text{Indirect effect of component } j \text{ via } i \text{ on } c}
 \end{aligned}$$

where c is the evaluated central node and x_c is its associated node value. The obtained number is an estimation of the “true amount” of risk attributable to the node and it can be utilized as an estimation of the risk in a future time point. A further modification of the aggregation procedure could be to specify the weight $v(c)$ as 1 instead of the value calculated by considering the network structure and the weights of the links ending at c . The main motivation for this is that, in general, we would not like the new estimation to be smaller as a consequence of a node being largely interconnected to other nodes of the network. For a node that has a lot of connection to other nodes with low risk level, the original formula would overemphasize the interconnectedness, and as a consequence the individual risk level, even from a very high starting value, could significantly decrease. In this case it is possible that the above formula results in an estimation greater than one. For this reason, when applying this modified weight when estimating the level of risk associated to a node, the final value of RiskRank should be calculated as $\min(RR_c, 1)$.

The proposed measure can be extended to account for more complex interaction effects. In the above discussion, paths with length not greater than 2 were considered as the potential set of connections affecting the analyzed node. By modifying the formula and considering paths with length greater than

2, we can account for indirect effects that may take into consideration the speed of the risk spreading throughout the system. By accounting for indirect effects from nodes that can reach the central node on a path with length at most k , one can estimate the level of risk by adjusting for a longer future time period. In this respect, the aggregated systemic risk obtained in the example evaluates a situation that will take place in a future time-point; if we assume that the delay of spreading risk from one node to a neighboring node is one time unit, then the example estimates the development of the system in two time units from now. Formally, to estimate the state of the system k time units from now based on the network representation at time point t , a different RiskRank measure can be defined by considering non-zero interaction values for nodes along a paths that have length not greater than k and has the node for which we are estimating the risk level as the endpoint of the path. Based on each measure we can obtain an estimation of the risk values in different nodes of the network at any point between now and k time units later. As in the case of paths of length 2, an even more important problem here is to define the interaction values to combine the value associated to the edges on paths with different lengths. In the original formulation of Choquet integral, this can be done by specifying a fuzzy measure μ_t^k to be a k -additive monotone measures described in [14] and moving beyond the complexity of the 2-additive Choquet integral. For instance, considering the product of values as the final path value for large k 's in general results in low effect values meaning that after a point we do not gain any new information by increasing the possible length of considered paths, while using the maximum of the individual values on the path would increase the systemic risk value significantly after every step. It always depends on the understanding of the underlying domain to determine for how many steps it is still meaningful to forecast based on the present situation. In highly fluctuating and rapidly changing systems the recommended value should be lower than in rather stationary systems.

4. RiskRank: An application to Europe

This section illustrates the use of RiskRank to aggregate risk in a European setting. It is worth noting that RiskRank does not require specific definitions of “links” and “individual risk”, but is rather open to any definition of the two measures in order to arrive at a final combined aggregate. The application shown herein aims at a final target of a European aggregate. The application provides aggregations to higher level in the hierarchy (from individual countries to Europe), and accounts for the interconnections in measuring risk at the same level. To test for the added value of RiskRank relative to only measuring individual risk, we show in the following performance comparisons in out-of-sample tests. We first describe the out-of-sample exercises and measures, and then move to the application.

4.1. Evaluating model performance

To judge the extent to which one measure outperforms another in a realistic setup, we need sufficiently sophisticated and carefully designed exercises and metrics. The evaluation exercises need to both measure the quality of signals when applied a realistic setup and measure performance with measures that mimic the problem at hand. This paper uses recursive real-time out-of-sample tests assess performance. In practice, this implies the use of a recursive exercise that derives a new model at each quarter using only information available up to that point in time. By accounting for publication lags and using information in the manner of an increasing window, this enables in our cases testing whether a measure would have provided means for predicting the global financial crisis of 2007–2008, and how measures are ranked in terms of performance for the task.

Following the standard evaluation framework for early-warning models in Sarlin [39], we aim at mimicking an ideal leading indicator $C_n(h) \in \{0, 1\}$ for observation n (where $n = 1, 2, \dots, N$) and forecast horizon h . This implies nothing else than a binary indicator that is one during vulnerable periods and zero otherwise. For detecting events C_n , we need a continuous measure indicating membership in a vulnerable state $p_n \in [0, 1]$, which is then turned into a binary prediction B_n that takes the value one if p_n exceeds a specified threshold $\tau \in [0, 1]$ and zero otherwise. The correspondence

between the prediction B_n and the ideal leading indicator C_n can then be summarized into a so-called contingency matrix that assigns every classification into one of four classes: true positives (TP, correct signals times of crisis), true negatives (TN, correct silence in tranquil times), false positives (FP, false alarms) and false negatives (FN, missed crisis). In terms of the elements of the contingency matrix, we can differentiate between two different types of classification errors that a decision maker may be concerned with: missing crises and issuing false alarms. To formulate the concepts of usefulness and relative usefulness as measures of classification performance in Sarlin [39], we define type I errors as the share of missed crises to the frequency of crises, i.e. $T_1 = FN/(FN + TP)$, and type II errors as the share of issued false alarms to the frequency of tranquil periods, i.e. $T_2 = FP/(TN + FP)$. Further, we need two terms: policymakers' relative preference between type I and II errors (μ) to account for the potentially imbalanced costs of errors and the unconditional probabilities of crises P_1 and tranquil periods P_2 to account for the potential difference in the size of the two classes. Based on these values, we can define the loss function as:

$$L(\mu) = \mu T_1 P_1 + (1 - \mu) T_2 P_2.$$

Further, based on this loss function, the absolute usefulness of the prediction model can be specified by comparing it to using the best guess of a policymaker (always or never signaling depending on class frequency and preferences):

$$U_a(\mu) = \min(\mu P_1, (1 - \mu) P_2) - L(\mu).$$

Finally, we compute relative usefulness, U_r to compare the absolute usefulness of the model to the absolute usefulness of a model with perfect performance ($L(\mu) = 0$). Additionally, to assess predictive performance, we also calculate standard measures from the classification and machine learning literature, in particular the area under the receiver operating characteristic (*ROC*) curve (*AUC*). These techniques provide both measures tailored to the preferences of a policymaker as well as more general-purpose measures to assess model performance. Other performance measures to be used in assessing the model include: (i) precision of signals $TP/(FP + TP)$, i.e. the share of correct signals to the frequency of signals; (ii) precision of tranquil predictions $TN/(FN + TN)$, i.e. the share of correct silence in tranquil times to the frequency of predicting tranquil time; (iii) recall of signals $TP/(FN + TP)$, i.e. the share of correct signals to the frequency of crisis times; (iv) recall of tranquil predictions $TN/(FP + TN)$, i.e. the share of correct silence in tranquil times to the frequency of tranquil times; (v) accuracy of the model $TP + TN/(FN + FP + TN + TP)$, i.e. the share of correct classifications.

4.2. RiskRank for Europe and its countries

This section measures systemic risk for individual European countries as well as aggregates to a pan-European level. In this application, we focus on individual risk as measured with country-level macro-financial imbalance indicators and real linkages as measured with real cross-border linkages. After describing the underlying data and models, we herein both evaluate model performance and exemplify model output with and without RiskRank.

To be able to compute RiskRank, we need to build a network of countries that includes both node and link values. We hence specify risk levels (probabilities x_c) for countries, and also measures of interconnectedness between all pairs of nodes (interlinkages $I(c_i, c_j)$). The approach for computing individual risk for each economy follows Holopainen and Sarlin [18]. To derive individual risk with early-warning models, we need crisis events and vulnerability indicators. The crisis events are based upon the IMF database by Laeven and Valencia [24]. The vulnerability indicators used include most common measures of widespread imbalances, such as excessive credit growth, excessive increases in stock and house prices, GDP growth, loans to deposits and debt service ratio, as well as more structural indicators, such as government debt, current account deficits and inflation. We use a standard logistic regression with 14 macro-financial indicators for countries and a forecast horizon of 5–12 quarters prior to crisis events, as is common in the literature. The network dimension is measured with BIS International Banking Statistics in terms of foreign claims of banking sectors on other economies. From the perspective of systemic banking crises, this provides ample means to capture the spread of

vulnerabilities across borders through real linkages. Yet, these are only one means to measure both the cyclical and cross-sectional dimensions of systemic risk and hence obviously also come with a number of deficiencies. It is hence worth noting that the models and data are given and the key focus is hence on relative performance of RiskRank versus standalone early-warning models.

To start with, we first estimate individual risk and RiskRank for each economy separately. In the case of RiskRank, this involves aggregating at the same level of hierarchy by also accounting for interconnectedness, whereafter we evaluate the performance of the individual risk model and RiskRank vis-à-vis the crisis event database. This provides the results in Table 1. As has been pointed out in several works (e.g., Sarlin [39], Betz et al. [5]), a feature of samples with imbalanced classes is that one needs to be more concerned of missing crises for models to be Useful. Given preferences μ , a pairwise comparison shows that the performance of RiskRank is never worse than the traditional individual risk model and it is better for preferences $\mu \in [0.3, 1]$. Figure 1 provides an example of model output for Germany with the individual model and RiskRank. The risk level is decomposed into individual risk, direct and indirect effects of other countries in case of individual countries. As we can observe from the figure, indirect effects are most often negligible compared to the other components, and while individual risk usually dominates the aggregated value, especially in crisis periods the direct impact from connected countries can significantly increase the risk level. More specifically, the timing of crisis signals come at a much earlier stage when accounting for direct and indirect effects. This represents the fact that domestic imbalances were fairly modest, while Germany was highly interlinked to countries with large vulnerabilities. As an early indication of an impending crisis, the figure illustrates the importance to incorporate vulnerabilities descending from “neighboring” countries, in addition to those building up in the domestic economy. Likewise, one can observe that the latest increases in risk are on the other hand related more to domestic imbalances, which again points to the potential need for and type of domestic policy actions.

Table 1: Signaling performance of individual models and RiskRank for countries with forecast horizon 5–12 quarters prior to crisis events.

μ	Individual		RiskRank	
	$U_r(\mu)$	AUC	$U_r(\mu)$	AUC
0.0	0 %	0.915	0 %	0.921
0.1	-6 %	0.915	-6 %	0.921
0.2	-3 %	0.915	-3 %	0.921
0.3	6 %	0.915	7 %	0.921
0.4	12 %	0.915	18 %	0.921
0.5	15 %	0.915	38 %	0.921
0.6	25 %	0.915	39 %	0.921
0.7	44 %	0.915	54 %	0.921
0.8	60 %	0.915	66 %	0.921
0.9	73 %	0.915	74 %	0.921
1.0	0 %	0.915	0 %	0.921

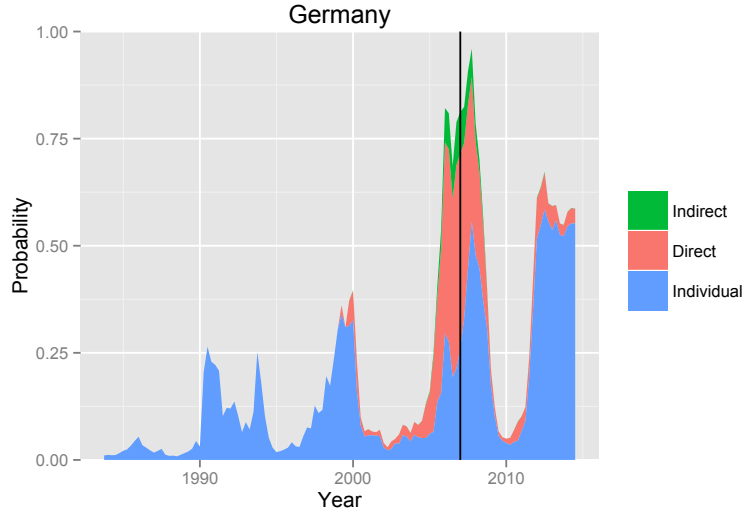


Figure 1: Examples of individual risk and RiskRank for Germany and Europe

RiskRank also allows to measure vulnerability at a European level by aggregating the country-specific measures. Figure 2 provides an example of model output for Europe with the individual model and RiskRank. In the figure, the European risk level is decomposed into direct and indirect impact of countries (red and blue stacks), whereas the yellow line shows the weighted average of the individual risk of countries and the black line the proportion of countries being in a pre-crisis state in a given quarter. Thus, the figure particularly depicts the increase in RiskRank when connected countries exhibit larger individual Risk (blue and red stacks vis-à-vis yellow line). Likewise, the figure also shows that unconditional probabilities (black line) do exceed a simple weighted average of individual probabilities, which indeed also points to the need for an aggregation function accounting for other factors.

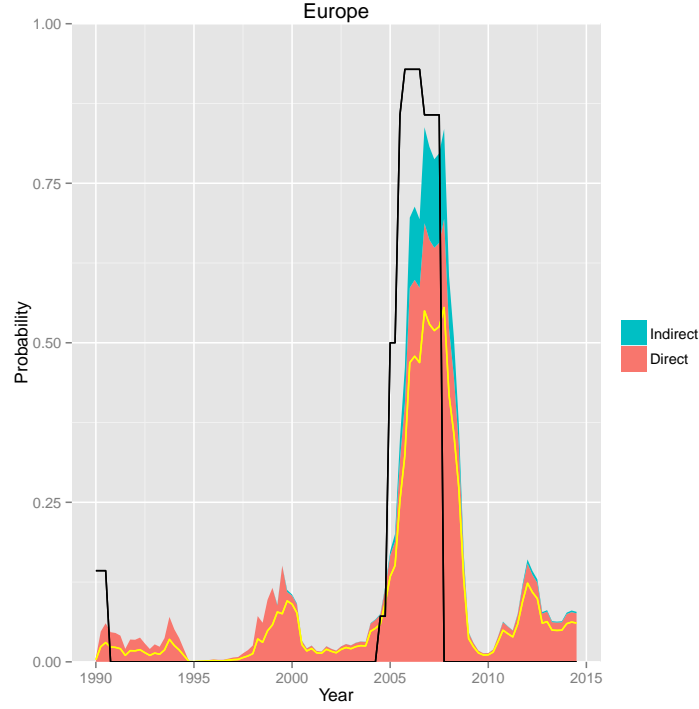


Figure 2: RiskRank for Europe with the yellow line representing the weighted average of the individual risk of countries, while the black line depicts the proportion of countries being in a pre-crisis state in a given quarter

4.3. Robustness of RiskRank

For a more detailed comparison of predictive performance, Tables 2 and 3 lists a range of previously introduced performance measures for the two above models. Further, we test the robustness of the above exercise when varying model specification. To check the sensitivity of the aggregation procedure with respect to the forecast horizon, we perform the analysis with two other scenarios: (i) 5–8 and (ii) 5–16 quarters prior to crisis events. As can be seen in Tables 4 and 5, RiskRank still results in higher AUC values and relative usefulness compared to the individual risk model. Naturally, the wider the forecast horizon is, the worse the predictions are, but the aggregated risk level used in RiskRank provides a more accurate estimation with clear improvements for higher values of the preference parameter μ (above 0.3).

Table 2: Performance measures based on individual probabilities

μ	TP	TN	FP	FN	Precision (C)	Recall (C)	Precision (T)	Recall (T)	Accuracy
0.0	0	1146	1	139	0.00	0.00	89.18	99.91	89.11
0.1	0	1146	1	139	0.00	0.00	89.18	99.91	89.11
0.2	0	1146	1	139	0.00	0.00	89.18	99.91	89.11
0.3	30	1138	9	109	76.92	21.58	91.26	99.22	90.82
0.4	30	1138	9	109	76.92	21.58	91.26	99.22	90.82
0.5	30	1138	9	109	76.92	21.58	91.26	99.22	90.82
0.6	98	1052	95	41	50.78	70.50	96.25	91.72	89.42
0.7	113	1028	119	26	48.71	81.29	97.53	89.63	88.72
0.8	116	1018	129	23	47.35	83.45	97.79	88.75	88.18
0.9	121	997	150	18	44.65	87.05	98.23	86.92	86.94
1.0	139	0	1147	0	10.81	1.00	-	0.00	10.81

Table 3: Performance measures based on aggregated probabilities

μ	TP	TN	FP	FN	Precision (EW)	Recall (EW)	Precision (T)	Recall (T)	Accuracy
0.0	0	1146	1	139	0.00	0.00	89.18	99.91	89.11
0.1	0	1146	1	139	0.00	0.00	89.18	99.91	89.11
0.2	0	1146	1	139	0.00	0.00	89.18	99.91	89.11
0.3	30	1138	9	109	76.92	21.58	91.26	99.22	90.82
0.4	30	1138	9	109	76.92	21.58	91.26	99.22	90.82
0.5	45	1127	20	94	69.23	32.37	92.30	98.96	91.14
0.6	114	1055	92	25	55.34	82.01	97.69	91.98	90.90
0.7	114	1055	92	25	55.34	82.01	97.69	91.98	90.90
0.8	114	1055	92	25	55.34	82.01	97.69	91.98	90.90
0.9	122	998	149	17	45.02	87.77	98.33	87.01	87.09
1.0	139	0	1147	0	10.81	1.00	-	0.00	10.81

Table 4: Signaling performance of individual models and RiskRank for countries with forecast horizon 5–8 quarters prior to crisis events.

μ	Individual		RiskRank	
	$U_r(\mu)$	AUC	$U_r(\mu)$	AUC
0.0	0 %	0.927	0 %	0.935
0.1	-11 %	0.927	-11 %	0.935
0.2	-5 %	0.927	-5 %	0.935
0.3	-3 %	0.927	-3 %	0.935
0.4	-2 %	0.927	1 %	0.935
0.5	4 %	0.927	7 %	0.935
0.6	11 %	0.927	16 %	0.935
0.7	22 %	0.927	34 %	0.935
0.8	47 %	0.927	55 %	0.935
0.9	70 %	0.927	76 %	0.935
1.0	0 %	0.927	0 %	0.935

Table 5: Signaling performance of individual models and RiskRank for countries with forecast horizon 5–16 quarters prior to crisis events.

μ	Individual		RiskRank	
	$U_r(\mu)$	AUC	$U_r(\mu)$	AUC
0.0	0 %	0.861	0 %	0.866
0.1	-5 %	0.861	-5 %	0.866
0.2	-2 %	0.861	-2 %	0.866
0.3	5 %	0.861	5 %	0.866
0.4	9 %	0.861	9 %	0.866
0.5	11 %	0.861	15 %	0.866
0.6	25 %	0.861	30 %	0.866
0.7	38 %	0.861	41 %	0.866
0.8	50 %	0.861	50 %	0.866
0.9	48 %	0.861	46 %	0.866
1.0	0 %	0.861	0 %	0.866

5. Conclusion

Estimating systemic risk is considered nowadays as one of the most prominent tasks as a consequence of the recent financial crisis. In this paper we introduced a new approach for assessing systemic risk motivated by advances in the domains of aggregation operators and network analysis. In the proposed approach, we combine risk indicators of entities in the financial markets, consequently modelling cyclical and cross-sectional systemic risk using our approach. As the result of an aggregation process, we obtain not only the aggregated risk levels of individual entities in a system and also the system as a whole, but the risk is decomposed into individual, direct and indirect components. While the proposed approach is motivated by the theory of aggregation operators and network analysis, it is closely related to, and from some perspectives expands upon the Shapley-index approach frequently used in economics, specifically in systemic risk analysis.

The approach is exemplified using the case of estimating systemic risk in a European setting. In the example, we provide an estimation of systemic risk from country-level risk and cross-border linkages. The out-of-sample performance of the approach in this case illustrates how aggregating risk levels in the network representation can improve on a traditional estimation method. This paper is to be extended with also bank-level results, where similar aggregation procedures allow estimating upward in the hierarchy from bank-level individual risk and linkages.

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